7. Complexity

> Of course I was attacked, from all around. "Don't you think that order can come from chaos?"
> "Uh, well, as a general principle, or…" I didn't understand what to do with a question like "Can order come from chaos?" Yes, no, what of it?
>
> Richard Feynman
> *Surely You're Joking, Mr. Feynman!* 1985

Describing the process of innovation as a nonlinear interaction between technology-push and requirements-pull is but one framing of a larger philosophical question: What are the foundations and methods of scientific inquiry? Two general processes are first, starting from a comprehensive whole and breaking it down to its particulars, and second, starting with the particulars and building towards a comprehensive whole. On the one hand, requirements-pull proceeds from the general to the specific and is related to reason, deduction, analysis, and differentiation. For example, the requirement for air superiority is a general concept that can be expressed by many arrangements of particular technologies, the selection of which depends in part on the attitudes and culture of the decision maker. On the other hand, technology-push proceeds from the specific to the general and is related to empirics, induction, synthesis, and integration. For example, independent technologies such as the jet engine, airframe structures, electronics, and ordnance can be integrated into a system that expresses the air superiority concept. The inductive progression builds up to a concept by relating observed elements.

Concept Building

The inductive approach to science may be represented by Francis Bacon, who rejected Copernican astronomy when empirical evidence seemed to fit Ptolemaic epicycles just fine.[1] The deductive approach may be represented by René Descartes, who rejected empiricism due to a "deception of the senses"[2] but failed to explain how distinct truths, like mathematical axioms (e.g., if A = B, then B = A), became self-evident points from which to reason in the first place.[3] While Bacon relied on empirical evidence found through experimentation, Descartes relied on pure reason independent of experience found in mathematics and logic. Though an interplay between the two methods was considered perhaps as long ago as Aristotle, it continued to be debated into the 20th century.[4] Alfred North Whitehead described the differences between induction and deduction, arguing that natural sciences were not the "rigid method" of induction alone, as Francis Bacon believed; science instead

---

[1] Hayek, Fredrich A. (1955). The Counter-Revolution of Science. New York, NY: First Free Press.

[2] Rene Descartes, Discourse on Method, Trans. L. J. Lafleur. (Bobbs-Merrill, 1950), pp. 5-7.

[3] (2017, July 6). "Rationalism vs. Empiricism." Stanford Encyclopedia of Philosophy. https://plato.stanford.edu/entries/rationalism-empiricism. The problem of Descartes' logic has been called the Cartesian Circle.

[4] Ailman, Christy. (2013). Philosophy of Math: The Beginnings of Mathematical Deduction by Induction. Azusa Pacific University. Retrieved from https://philarchive.org/archive/AILMDB-2.

required the deductive reasoning from mathematics to verify the internal consistency of its conceptual system. "What Bacon omitted," Whitehead wrote,

> "… was the play of a free imagination, controlled by the requirements of coherence and logic. The true method of discovery is like the flight of an aeroplane. It starts from the ground of particular observation; it makes a flight into the thin air of imaginative generalization; and it again lands for renewed observation rendered acute by rational interpretation."[5]

Structuring ideas from observations, followed by an unstructuring, restructuring and restructuring again is the basic method of learning. By contrast, the systems analysis approach expressed in the Department of Defense is largely deductive in nature. Indeed, the very term "systems analysis" invokes deductive as opposed to inductive methods. "Rather than waiting upon experience in the real world," Aaron Wildavsky explained, "the [systems] analyst tries various moves in his model and runs them through to see if they work."[6] Systems analyses sprung out of operations research in WWII, which generally had well-defined objectives amenable to mathematical tools such as linear programming and queuing theory. While the objectives are given and assumptions about the environment specified in operations research, Alain Enthoven and others explained that a major task of systems analysis was defining objectives and operating assumptions.[7] E.S. Quade observed that systems analysts are

> "… likely to be forced to deal with problems in which the difficulty lies in deciding *what out to be done, not simply in how to do it…* The situation is not like an *empirical science*, which starts with observed facts, but more like that of *mathematics*, where the results take any 'validity' they might have in the real world from the assumptions… it is important that the assumptions be the 'right' assumptions."[8]

Quade clearly expressed the deductive methods of systems analysis and contrasted it to empiricism. Nevertheless, Quade could not put systems analysis on the firm foundations of reason because "judgment and intuition permeate" the models, particularly when framing the goals and assumptions.[9] It is no surprise, then, that systems analysts largely defended weapon requirements rather than the feasibility of certain technologies.[10] At the center of systems analysis lies the conceptual model from which technical solutions are reasoned from. One problem that James Schlesinger commented on is that "our ability to formulate models depends upon our knowledge of the mechanics of the real world."[11] Admiral Rickover complained how systems analysts "have little or no scientific training or technical expertise… Their studies are, in general, abstractions. They read more like the rules of a

---

[5] Whitehead, Alfred North. (19XX). Process and Reality.

[6] Wildavsky, Aaron. (1966, December). "The Political Economy of Efficiency: Cost-Benefit Analysis, Systems Analysis, and Program Budgeting." *Public Administration Review*, Vol XXVI, No. 4.

[7] Murdock, Clark A. (1974). *Defense Policy Formation: A Comparative Analysis of the McNamara Era*. Albany, State University of New York Press, pp. 46.

[8] Roherty, James R. (1970). *Decisions of Robert S. McNamara: A Study of the Role of the Secretary of Defense*. Coral Gables, FL: University of Miami Press, pp. 85-86.

[9] Quade, E.S. (1966, March). "Systems Analysis Techniques for Planning-Programming-Budgeting." RAND Corp., P-3322.

[10] Borklund, C. W. "Cost-Effectiveness' vs. Creativity: Part 2, A "Wait-and-See" Philosophy Can Squelch Initiative." *Armed Forces Management*, September 1967, pp. 57 – 59.

[11] Schlesinger, James R. "Uses and Abuses of Analysis." Published in Jackson Committee Hearings, "Planning, Programming, Budgeting: Inquiry of the Subcommittee on National Security and International Operations." U.S. Government Printing Office, Washington: 1970.

game of classroom logic than a prognosis of real events in the real world."[12] Though critics appreciated the logic and rigor inherent to systems analysis, they repeatedly pointed to disconnects from empiricism, from knowledge gained by trying things out. Representative Porter Hardy Jr. provided a similar assessment during an appropriation hearing in May 1968. "My best information," he said, "is that there are no significant military inputs into these analyses."[13]

John Boyd, hero of the lightweight fighter program, also struggled with the systems analysis approach dominant in DoD decision making. Yet Boyd's experience belies the subtleties of its implementation. When evaluating the designs of the F-X project in 1966, Boyd criticized the people and institutions who "wormed their pet technologies into the final design." Instead of valid technical features emerging from the requirements, Boyd found that the F-X requirements were altered to fit the desired technical features.[14] Even the flailing F-111 may not have been a product of pure systems analysis. I.F. Stone reported that systems analysts at the OSD level wrote a memo critical of the F-111 early in its design phase but Enthoven rejected it "on the grounds that it would call down bureaucratic wrath on the fledgling systems analysis office."[15] Systems analysis may have led to successful designs if it's ideal remained uncorrupted by special interests. Schlesinger noticed how "Studies are driven by the underlying assumptions, and these may be imposed directly or indirectly from above… The role of analysis then becomes not so much to *sharpen* the intuitions of the decisionmaker as to *confirm* them."[16] For example, Boyd disapproved of the prevailing intuition of the bigger-higher-faster-farther aircraft. A speed of Mach 2.5 was only built into the F-X requirements because a new technology emerged, variable geometry inlets, despite the fact that the technology entailed severe design penalties to maintenance, cost, and range.[17] James Burton reported how Boyd believed that technical features are the output from a disciplined design trade-off, and not the input.[18] His design philosophy at the time appears true to the deductive method and aligned with the intent—though perhaps not the practice—of systems analysis. Yet as Boyd came to discover himself, his actual process of learning was more like that described by Whitehead, an interaction between inductive and deductive approaches.

Consider a sketch of Boyd's journey to the lightweight fighter. He first spent many years gaining experience as a fighter pilot and then classified all the air combat maneuvers he observed in "Aerial

---

[12] Alain C. Enthoven and K. Wayne Smith. (1971). *How Much Is Enough? Shaping the Defense Program, 1961-1969.* New York, NY: Harper & Row, Publishers, pp. 78.

[13] Poole, Walter. . History of Acquisition in the Department of Defense Volume II: *Adapting to Flexible Response, 1960-1968.* Historical Office, Office of the Secretary of Defense, Washington: 2013, pp. 31.

[14] Burton, James G. (1993). *The Pentagon Wars: Reformers Challenge the Old Guard.* Annapolis, MD: Naval Institute Press, pp. 14.

[15] Murdock, Clark A. (1974). *Defense Policy Formation: A Comparative Analysis of the McNamara Era.* Albany, State University of New York Press, pp. 165.

[16] Schlesinger, James R. "Uses and Abuses of Analysis." Published in Jackson Committee Hearings, "Planning, Programming, Budgeting: Inquiry of the Subcommittee on National Security and International Operations." U.S. Government Printing Office, Washington: 1970.

[17] Boyd, John. (1991, April 30). Impact of the Person Gulf War and the Decline of the Soviet Union on How the United States Does Its Defense Business. Committee on Armed Services, Hours, No 102-17, pp. 688.

[18] Burton, James G. (1993). *The Pentagon Wars: Reformers Challenge the Old Guard.* Annapolis, MD: Naval Institute Press, pp. 14. Similarly, Pierre Sprey derived the requirements for the A-10 by a detailed study of close air support missions performed by German aviators in WWII and U.S. aviators in Korea.

An F-15, formerly the F-X, showing its variable geometry inlets in two positions.

Attack Study." Boyd's schemata were so thorough that no major additions have been identified. Having so matched his classification system with experience, Boyd wondered what tied the maneuver-countermaneuver strategies together and whether it could be formulated differently.[19] By studying engineering at Georgia Tech, a broadened experience led Boyd to the useful real-world concept of entropy, which he applied back to air combat scenarios and formalized into mathematics with Energy-Maneuverability (EM) theory. The F-X design provided Boyd his first opportunity to apply EM theory to evaluate aircraft design. However, to deduce proper technical evaluations from EM theory, it first took several rounds of induction and deduction to build up to the EM theory concept. Concepts so aligned with reality do not arise from pure thinking alone or axiomatic "truths" such as "more speed is better."

The inductive-deductive cycle continued as inadequacies in EM theory were demonstrated during the flyoff competition between the YF-16 and YF-17 in 1974. While both planes were predicted on paper to have similar maneuverability, pilots gave a distinct advantage to the YF-16. EM theory certainly improved fighter aircraft evaluation but it was not yet a map of reality, calling for the structuring of an improved concept. Boyd, however, had moved on. Weary of the politics surrounding aircraft development, Boyd looked instead to apply his maneuverability concepts to more general topics including learning, human organization, and war. For the prior few years Boyd was keenly interested in a range of subjects, including epistemological debates on the theory of knowledge by luminaries such as Karl Popper, Michael Polanyi, and Thomas Kuhn. As debates raged in 1975 over whether the Air Force would inventory the F-16 and A-10, Boyd resigned his post. The next year, John Boyd released a short paper entitled "Destruction and Creation" which described the concepts that became the foundation of all of his subsequent work on the military sciences. He provided a justification that an inductive and deductive cycle is not just desirable for model building, but an inevitable fact of life.[20] Over the next two decades, Boyd refined and presented his ideas on maneuver warfare and the "OODA" loop. It will be shown how Boyd's work anticipated the interdisciplinary studies of complexity and nonlinear systems which contribute substantially to our understanding of economic systems and defense acquisition.

Destruction and Creation

Boyd's short 1976 paper, "Destruction and Creation," will be used as an introduction to a broader shift in both the natural and social sciences towards thinking in terms of complex adaptive systems. In this chapter and the next, these ideas will be applied to defense acquisition. Boyd went after a big idea in the paper, a general theory of how we create mental concepts that allows us to adapt to a

---

[19] Croam, Robert. (2002). *Boyd: The Fighter Pilot Who Changed the Art of War.* Robert Croam: USA, pp. 116.

[20] Spinney, Franklin "Chuck." (2014, December). Evolutionary Epistemology. Version 2.4. Retrieved from https://fasttransients.files.wordpress.com/2010/03/spinneyevolutionaryepistemology2-4.pdf.

changing environment and "improve our capacity for independent action." The ability to generate mental concepts and use them to decide upon real world actions is indeed what sets humans apart. For example, Schrödinger's Equation is an articulation of quantum mechanical concepts which became usefully applied to our understanding of technologies such as computers, GPS, and lasers. The relevance of technology to our survival need not be elaborated. We can say that survival depends on adaptation which, in the human world, need not take place in our genes but in our minds. Human adaptation in the world depends on decision-making about technologies in the broadest sense, an activity dependent upon underlying mental concepts. Boyd wondered, "How do we generate or create the mental concepts to support this decision-making activity?" The question also underpins systems analysis, where military survival necessitates decisions concerning the direction of technological progress. Mental concepts frame the assumptions about technologies and requirements which decide the course of resource investment.

Boyd then described the inductive and deductive approaches for building mental concepts described above. "Now keeping these two opposing idea chains in mind," one from the specific-to-general and the other general-to-specific, Boyd likened deduction to the "destruction" of a domain or concept into its many parts and likened induction to the "constructive" process of reconstituting the parts into new domains or concepts. So long as the reconstitution does not create the exact same relations among the parts—indicating creativity—new and different concepts have emerged. "Recalling that we use concepts or mental patterns to represent reality," Boyd wrote, "it follows that the unstructuring and restructuring just shown reveals a way of changing our perception of reality. Naturally, such a notion implies that the emerging pattern of ideas and interactions must be internally consistent and match-up with reality. To check or verify internal consistency, we try to see if we can trace our way back to the original constituents that were used in the creative or constructive induction." The tracing back from concept to parts without contradiction is related to reason and logic. Pending consistency, a new concept only becomes useful if it also presents us with new hypotheses and experimental tests, allowing for new observations and thus new concepts.

After many iterations of destructive deduction and creative induction, Boyd imagines how we may create a concept whose consistency and match-up with reality are so powerful that there is no further appeal to expand, complete, or modify the concept. This leads to an inward-oriented effort to make increasingly subtle observations that may improve the explanatory power of our concept. Boyd then suspected that at some point, anomalies or inconsistencies will appear from the inward-oriented application of deduction and induction. Any anticipated difference between new and subtler observations with previous observations "suggests we should expect a mismatch between the new observations and the anticipated concept description of these observations. To assume otherwise would be tantamount to admitting that the previous constituents and interactions would produce the same synthesis as any newer constituents and interactions." Subtler observations provide fodder for creatively synthesizing different and potentially more powerful concepts.

Boyd quickly introduced the idea of an ultimate concept requiring no further expansion or modification before he quickly cuts it down. No concept, he claims, can so completely describe the real world that we can consistently explain all observations. Boyd stated that "we should anticipate a mismatch between phenomena observation and concept description of that observation." As we shall see, the idea is important for human organization, whether military, economic or otherwise, because it implies the limits of planning; no centralized office can hold a complete concept which can be used to calculate optimal courses of action in all cases. Boyd supports the claim by integrating three notions: Gödel's Incompleteness Theorems; Heisenberg's Uncertainty Principle; and the 2nd Law of Thermodynamics (Entropy). Before elucidating the three notions and interpreting their relevance, an epistemological background will be provided, one that Boyd was largely aware of from his readings.

Colonel John R. Boyd retired from the Air Force in 1975 to pursue a wide-range of studies.

Positivism

Though most histories must start before the beginning, this brief overview will start with Isaac Newton who, in 1687, published *Principia Mathematica*. He found quantitatively precise laws of Nature in classical mechanics and the inverse-square rule for gravity. If a consistent and complete description of nature can be deduced from a finite set of quantitative laws, the logical conclusion is a scientific determinism. Pierre-Simon LaPlace conjectured that if all the positions and velocities of all the particles in the universe could be known, then the laws of Nature will allow a "vast enough" intellect to calculate all past and future states of the universe. Free will must have been an illusion. By the end of 19th century, many scientists believed they were reaching a complete description of natural laws in which they could theoretically describe and predict all aspects of our empirical world. As related in Chapter 4, the vaunted success of natural laws in prediction eventually inspired the German Historical School and the American Progressive movement, reshaping business organization and public policy based on systematic planning methods supposedly associated with science.

The positivist view also provides a compelling philosophical rationale for weapon systems analysis. If a systems analyst knew all the laws of physics, he could derive all feasible technological arrangements and choose the optimal course based on the military requirements involved. Technical solutions need not be derived from the crude and wasteful empirical method of trial-and-error. All solutions can be calculated from the natural laws underlying elementary parts, and each of which may be validated or refuted by an independent third-party. If our knowledge of natural laws were complete, literally every technology the future may hold can be planned today, even if it couldn't practically be accomplished. As a result, a small number of the brightest people—those with the best grasp of natural laws—could sit in the Pentagon and steer the course of defense technology, which has the added benefit that holistic as opposed to parochial requirements will balance the equations. The process provides a logical basis for weapons choice and cannot be refuted without challenging the requirements or, what might seem outrageous, the laws of physics. Systems analysis becomes a far humbler endeavor, however, if it turned out that our models of natural laws were either incomplete—they cannot determine all feasible technologies—or inconsistent—they may wrongly assess technical feasibility.

For Immanuel Kant in the 18th century, natural laws such as Newton's inverse-square rule for gravity are not a window into the real world, or "things in themselves." Instead, our understanding of natural laws structures the way things, or phenomena, appear to us. The reason mathematics is so effectively applied to our world, such as Newton's law of gravity, is because our perception of the world has been structured by that mathematics. Our minds are hardwired with geometry and arithmetic, so when we look at the world and order our surroundings, Kant argued that it is already structured spatially (geometrically) and temporally (arithmetically). We cannot experience a world that doesn't conform to our own geometry and arithmetic. Mathematics, in a sense, is the language in which we interact with the world of phenomena. Different geometries and arithmetics can correspond to different ways of structuring the world around us. Curiously enough, Kant believed that Euclidean geometry of flat planes was the last word on geometry, yet when Carl Gauss and others dropped parallel lines axiom a non-Euclidean geometry of curved space was created. The concept proved crucial to Albert Einstein's formulation of relativity in which it was discovered that the structure of space and time is in fact curved. Mathematics for Kant was not the "truth" or a line of communication to Plato's world of forms; humans brought mathematics into the world and it structures our view of phenomena, such as providing the mental lens in 1919 to see light bending due to the curvature of space-time.[21]

Kant set the agenda for debates on epistemology. Bertrand Russell observed that "Kant's inconsistencies were such as to make it inevitable that philosophers who were influenced by him should develop rapidly either in the empirical or in the absolutist [deductive] direction."[22] In fact, Russell himself and fellow mathematician Gottlob Frege were convinced earlier in their careers that Kant was wrong; they believed that mathematical truths were not of our own making. The question they wanted to resolve was whether people discovered mathematics or invented it, with implications for whether it was objectively true or not. Frege set out to put mathematics on a logical foundation by proving that set theory really belonged to logicism. To do so, numbers first had to be defined as sets where membership depends on meeting certain conditions. The number two, for example, was defined the set of all objects that had two members. A tank is the set of armored tracked vehicles with a gun turret; it includes the M4 Sherman, the T-34, the Panzer IV, and all other vehicles which fit the set described by "tank." These sets are like Boyd's domains or concepts which constitute various parts. And like Boyd who conjectured a complete concept that no new observation would contradict, Bertrand Russell conjectured the set of all sets. The paradox that Russell found led to him down a path that ended in his rejecting the reality of numbers and logical form, reducing mathematics to a useful but tautological exercise as opposed to a window into the nature of reality.

Self-Referential

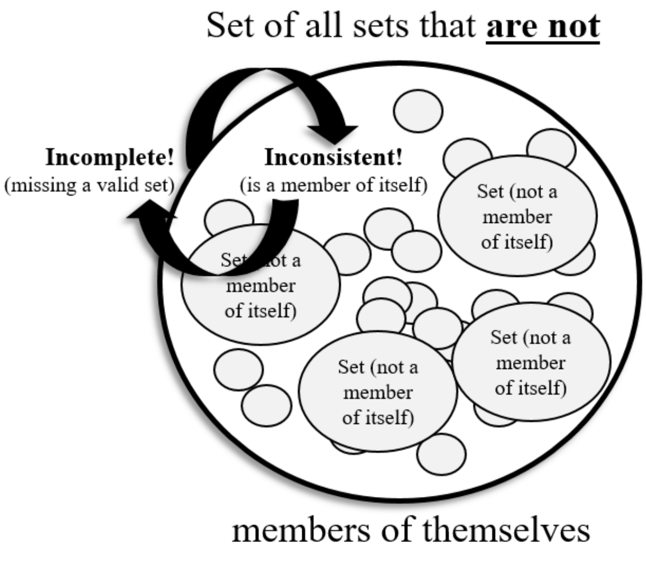
---

[21] Monk, Ray. (1996). *Bertrand Russell: The Spirit of Solitude 1987-1921.* London: Vintage.

[22] Russell, Bertrand. (1946). *History of Western Philosophy,* pp. 650.

In Gottlob Frege's set theory, there are two broad types of sets: those which are members of themselves and those which are not. The set "tank" is not itself a tank, it is a concept that includes all the sets of objects fitting its conditions (M4 Sherman, T-34, Panzer IV, and so forth). The concept of a tank itself does not bring armored firepower to the real world. Similarly, the number "2" does not itself have two members, and so is not member of its own set. Most sets are not members of themselves. However, "English" is a word within the set of English language words, and so is a member of itself. "Spanish" is not a member of itself, it is an English word and so wouldn't be in the set of Spanish words. The example shows how sets which are members of themselves are self-referential and hierarchical; English is an English word that describes itself.

One way of deriving a set that is a member of itself is to find the set of objects not described by another set. For example, the set of all sets that are not tanks includes ships, roses, chairs, and so forth. When we create the set of all things that are not tanks, we have created a new set and may ask, "Is this set of all non-tank sets itself, not a tank?" The answer, of course, is "yes," the set is not a tank, and therefore it is a member of itself. Otherwise, we would observe a valid set that meets the "not a tank" condition without finding it in our set. Our set would be incomplete. To avoid this discomforting fact, the set must be a member of itself. Similarly, when we form a set of all things that do not have two members, the resulting set also does not have two members, and so it must be a member of itself. Notice we are grouping into one set various other sets at different levels of a conceptual hierarchy. Self-membership within a set results from a self-reference of that hierarchy, such as asking whether the set of all sets meeting certain conditions itself meets those same conditions.



The set of all sets which are not members of themselves is represented by the outer circle. If it is a member of itself, it is inconsistent. If not, it is incomplete.

The reason for diverging down this path is that it leads to a paradox at the heart of mathematics as applied to logical form. Bertrand Russell wrote to Frege in 1903 asking about the complete concept: the set of all sets that are not members of themselves (including tanks and the number two but leaving out sets that are not tanks and are not two). Would this set be a member of itself or not? Well, if it really is the set of all sets that are not members of themselves, the set cannot include itself. But if it doesn't include itself, then it is missing the inclusion of a valid set that isn't a member of itself. It is incomplete. If we make the set a member of itself, then the set no longer meets the conditions for inclusion in the set of all sets that *are not* members of themselves. It is inconsistent. We arrive at a paradox. The set of all sets is either a member of itself or not, but that question cannot be decided. If it were decided, it would be as if, in a sense, the set is both "itself" and "not itself" at the same time. When we look into the set of all sets that are not members of themselves, it would be inconsistent if it included itself and incomplete if it were excluded. If the matter cannot be resolved, how can we trust any result from the mathematical system?

Frege immediately recognized that the whole of his work faced a serious challenge. He sought to put mathematics into logical form without encountering the vagaries and paradoxes of language. For example, language that is rich enough to talk about itself encounters inconsistencies, such as the "liar paradox," where the statement "This statement is false" is neither true nor false. Think about it. If we think the statement is true, and it's telling us it's false, then we ought to believe the statement and think that it is false. But wait, we started by thinking it was true! The "liar paradox" problem turned out not to be limited to language; Russell's paradox and others show how the problem falls onto mathematics as well.

Starting in 1900, David Hilbert, one of the most famous mathematicians of his day, sought to put mathematics on a solid axiomatic footing from which all propositions can be proved either true or false. Of twenty-three problems identified, two are problems about what can be proved by mathematics and can be summarized in three questions: Is mathematics consistent (only proves true statements)? Is it complete (proves all true statements)? Is it decidable (a definite procedure for every statement with results in finite time)? Hilbert's program was crucial not just for mathematics, but for logical positivism which viewed physics, and by extension all of sciences, as an application of mathematics. With a definite procedure for correctly proving all true statements, mathematics and the sciences could move towards finality. But if mathematics were inconsistent, incomplete, and/or undecidable, then it cannot be a fountain of discovery for all scientific truths. Such a result would also destroy the framework reasoned from by those in the Department of Defense who went headlong first into unification, and then into systems analysis and program budgeting.

Hilbert's program was thoroughly dashed in 1931 by a young man named Kurt Gödel. In essence, he demonstrably proved using arithmetic that arithmetic itself was either incomplete or inconsistent (later, Alan Turing proved it was undecidable). Gödel accomplished this feat—the first major result in logic since Aristotle—by generating a situation like the "liar paradox." The analogous statement Gödel mathematically employed is: "This statement is unproveable." If it is proved, the system is inconsistent. Otherwise, the system is incomplete. To make the self-referential statement mathematically, Gödel cleverly invented a way for mathematics to talk about itself. He imagined an enumerator that would codify every arithmetical function into a unique code number. The following will illustrate Gödel's proof.[23]

Incompleteness

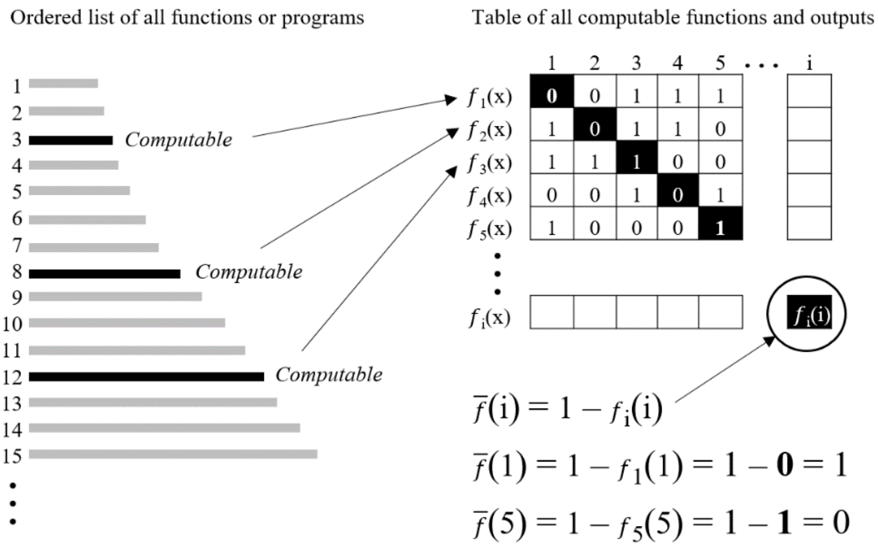
Imagine listing the code of every computer program possible in order of code length. Some short programs correspond to simple tasks and would be higher on the list. Other long programs will execute complex tasks and would be lower on the list. Many programs would amount to gibberish with no practical value but still they're ordered in the list. The point is that every possible computer

---

[23] UC Davis Academics. (2015, March 10). Gödel for Goldilocks: Gödel's First Incompleteness Theorem. https://www.youtube.com/watch?v=9JeIG_CsgvI.

program, representing every mathematical function, is fully enumerated in the list and ordered by length. Let's assume that from this list, we can collect the set of all computable programs, or functions, that takes in any positive integer "x" and outputs "$f(x)$" where the output is either a 1 or 0 (true or false). We now have a set of all computable functions (it appears complete) and for any input we can derive whether it is true or not (it appears consistent). Here is where the self-referencing comes into play. Let's define a new function: $\overline{f}(i) = 1 - f_i(i)$, where $f_i(i)$ is simply the output from the i^{th} function in our enumerated set when we input the i^{th} positive integer. So if the first function in our ordered set were $f_1(x)$ and we input the integer $x = 1$, suppose the output were 0. Our new function would then equal $(1 - 0)$ or 1. And if the fifth function in our ordered set were $f_5(x)$ and inputting $x = 5$ gave the output of 1, then our new function $\overline{f}(5) = 1 - f_5(5) = 1 - 1 = 0$. In other words, the new function will output the opposite value from different functions enumerated in our set, the choice of which depends on the input value. Briefly, we have shown that our new function, $\overline{f}(i)$, provides a valid output of 1 or 0 (true or false), but that output cannot be found anywhere in our enumerated set of computable functions. We have created a statement which we can recognize to be valid, but it cannot be derived within the standard axioms of set theory. Not all true statements are provable in the system because we have enumerated the entire system and still cannot find the answer. This is Gödel's First Incompleteness Theorem: Any consistent formal system *F* within which a certain amount of elementary arithmetic can be carried out is incomplete. Gödel then went further with his Second Theorem: The consistency of any consistent system *F* cannot be proved in *F* itself. Even if arithmetic were consistent, we cannot prove its consistency by the axioms that comprise it. As John Boyd explained in "Destruction and Creation":

> "Such a result does not imply that it is impossible to prove the consistency of a system. It only means that such a proof cannot be accomplished inside the system. As a matter of fact since Gödel, Gerhard Gentzen and others have shown that a consistency proof of arithmetic can be found by appealing to systems outside that arithmetic. Thus, Gödel's Proof indirectly shows that in order to determine the consistency of any new system we must construct or uncover another system beyond it. Over and over this cycle must be repeated to determine the consistency of more and more elaborate systems."



Gödel incompleteness illustration. All computable functions are ordered in a list and we can generate a table of all possible states of all computable functions. We can recognize a particular function is a valid part of our system, but it cannot be found in our enumerated table.

Indeed, consistency and completeness at one level of mathematics can be proved by appealing to higher levels of mathematics so long as the former is a strict subset of the latter. The reason in a nutshell is that if you have problems emanating from self-referencing statements, uncovering a more powerful system again fixes your point of reference so you are not trying to "look into" your own system from the inside. Such a problem occurs when you think about thinking. It also occurs when you take the systems analysis approach. The analyst's formal model requires talking about itself, talking about sets of technologies and missions and requirements, where each member has quantifiable attributes that can be related. In jargon, it requires second-order logic such as found in the statement "every set of requirements has an optimal technical solution" whereas first-order logic is found in the statement "every requirement has a technical solution." It is worthy of note that Gentzen only proved the consistency and completeness of arithmetic in full for first-order logic, not second-order logic.[24] Yet systems analysts care about finding better or best decisions, not any valid decision. It requires self-referential statements as to what is best for "national security." As we observe, sometimes the answer to defense questions are found by appealing to a larger system, or national policy objectives.

Going back to Boyd, we have a concept—a formal system—that we use for decision-making. If the concept is in fact consistent and rich enough to talk about itself, then by Gödel's First Theorem it must be incomplete. If the concept is incomplete, then Boyd argues that we must expect a mismatch between concept description and real-world observations. There is no room for the unexplained fact in a domain covered by a complete theory. Boyd only applied Gödel to a person's conceptual model; it is incomplete. He was not reasoning by analogy by applying Gödel incompleteness to a person's range of action or observation. If valid, it follows that when we interact with a larger and more powerful system, namely the universe, we will inevitably find *surprise and novelty*. This does not mean we can never completely describe all the laws of Nature; rather, we cannot predict all phenomena emanating from them in a well-defined system. In mathematics, the problem arises when statements cannot be proved from the axioms. It could be a result of Gödel incompleteness, or it could be that we have not been clever enough to derive the proof. For example, the Goldbach conjecture states that every even integer greater than two is the sum of two primes. We have never found a case where

---

[24] Alonzo Church and Alan Turing proved that first-order logic was, nevertheless, undecidable.

the statement is not true, but it hasn't been proven. Is it unprovable due to Gödel (there is no proof), or is the proof out there for some imaginative mathematician to find? In the sciences, our inability to predict all real-world phenomena seems, at this point, to largely result from the inadequacy of our current theories of Nature. Mathematician John D. Barrow commented that the state of physics "would become more like Gödel incompleteness if we could find no extension of the theory that could predict the new observed fact."[25] For example, relativity and quantum mechanics are powerful concepts in their own domains, but they appear to explain phenomena at different scales and thus appear inconsistent within a single system. Physics has yet to reach a unified theory where Gödel incompleteness may rear its head.

Another difference is that scientists cannot choose any logical system they'd like, unlike mathematicians who may simply assume a system of axioms and deductive laws. The scientific method operates from the reductionist assumption that if we can observe and predict the functioning of the elementary parts, then we can understand all higher levels of phenomena. It calls for controlled experiments that isolate cause and effect. The goal is to unify the different scientific accounts of phenomena at different scales. To first reach a unified theory of Nature, the we need to understand the dynamics of elementary particles. Otherwise, our predictions would be lacking a major ingredient of causality. To test alternative conjectures and discover which ones yield accurate predictions, we must gather information on the initial conditions. This requires us to make increasingly precise observations. However, we cannot make arbitrarily precise predictions at very small scales without first refuting Heisenberg's Uncertainty Principle which, briefly, finds that information on initial conditions cannot exist regardless of the precision of our measurements. Only a statistical description of possible futures can be made. Both Gödel and Heisenberg imply indeterminacy is built into our mental and physical worlds.

Uncertainty

Werner Heisenberg's 1927 results have enjoyed a long history of successful replication across a wide range of application without contradiction. As Boyd explained, we cannot "simultaneously fix or determine precisely the velocity and position of a particle or body… Examination of Heisenberg's Principle reveals that as a mass becomes exceedingly small the uncertainty or indeterminacy, becomes exceedingly large." In other words, even if there existed an intellect "vast enough" to compute all past and future states of the universe, it could never collect the initial conditions on position and velocity for even a single particle with which to make that computation. If it wanted to know the position of a particle with arbitrary accuracy, then by Heisenberg, the intellect could no longer know the precise velocity, and vice versa. Both values are simultaneously required for point predictions of the particle's future. One common analogy is to consider a table of rows and columns. Finding what row a particular value is in tells us nothing about what column it is in, and if we scroll up to discover the column, we lose track of value's row. Like Heisenberg, Boyd attributed the effect to the influence of an observer. When we attempt to measure an electron by a microscope, the accuracy is limited by

---

[25] Barrow, John D. (2006). "Gödel and Physics." Published in *Kurt Gödel and the Foundations of Mathematics: Horizons of Truth*, eds. M. Baez, C. Papadimitriou, H. Putnam, D. Scott and C. Harper, pp. 255-276, Cambridge UP, (2011).

the wavelength of light employed. As we shorten the wavelength to more precisely determine the position of the electron, we are also increasing the energy of the light which disturbs the electron. Heisenberg described the consequences from the effect called Compton scattering:

> "At the instant of time when the position is determined, that is, at the instant when the photon is scattered by the electron, the electron undergoes a discontinuous change in momentum. This change is the greater the smaller the wavelength of the light employed, i.e., the more exact the determination of the position. At the instant at which the position of the electron is known, its momentum therefore can be known only up to magnitudes which correspond to that discontinuous change; thus, the more precisely the position is determined, the less precisely the momentum is known, and conversely."[26]

When we ask the question of position, the electron scatters and we lose the ability to simultaneously ask precise questions of velocity. Heisenberg originally suggested that the electron really has a definite position and velocity, but we run into a practical problem of measuring that fact because we disturb the system with our measurement.[27] In short, we are ignorant of reality. Though Heisenberg abandoned the deterministic view, it was later retained by Bohmian mechanics[28] which found that particles have precise positions at all moments. However, particle velocities (and therefore trajectories) are determined by a "pilot wave" whose value depends simultaneously on all other particles (they share a "universal wave"). Any quantum experiment in a closed system must include an observing apparatus A whose pilot wave interacts with the observed phenomena P. A bit more technically, if P were decoupled from the apparatus A, then P would be guided by a pilot wave with a definite velocity that obeys Schrödinger's linear, and therefore predictable, equation. When we place apparatus A into P's closed system (creating the larger system A + P) particle P's pilot wave is conditional on the pilot wave of apparatus A. In other words, when we attempt to observe the particle's position our presence influences the particle's velocity described by the pilot wave. Because our observing apparatus cannot measure itself, we cannot predict the deterministic path of the particle. In Bohmian mechanics, when we attempt to more precisely measure the position, our apparatus becomes an increasingly important part of the combined system, A + P, and affects the particle's velocity. If we accept that Heisenberg's Principle "implicitly depends upon the indeterminate presence and influence of an observer," Boyd argued that "the magnitude of the uncertainty values represent the degree of intrusion by the observer upon the observed." We return the self-referencing problem.

---

[26] Heisenberg, Werner. (1927, March 23). "Uber den Anschaulichen Inhalt der Quantentheoretischen Kinematik und Mechanik." *Zeitschrift f̈ur Physik* 43 (1927), 172–198. English translation in J.A. Wheeler, H. Zurek (eds.). *Quantum Theory and Measurement*, Princeton Univ. Press, Princeton, 1983, 62–84.

[27] Heisenberg, Werner. (1930), *Physikalische Prinzipien der Quantentheorie(in German), Leipzig: Hirzel* English translation The Physical Principles of Quantum Theory. Chicago: University of Chicago Press, 1930.

[28] For more on De-Broglie-Bohm (Bohmian) mechanics, see: (a) Wigner, Eugene P., 1976, "Interpretation of Quantum Mechanics", lecture notes; revised and printed in Wheeler and Zurek 1983: 260–314. (b) Dürr, Detlef, Sheldon Goldstein, and Nino Zanghì, 2009, "On the Weak Measurement of Velocity in Bohmian Mechanics", *Journal of Statistical Physics*, 134(5): 1023–1032. (c) Dürr, Detlef, Sheldon Goldstein, and Nino Zanghì, 1997, "Bohmian Mechanics and the Meaning of the Wave Function", in R.S. Cohen, M. Horne, and J. Stachel (eds), *Experimental Metaphysics—Quantum Mechanical Studies for Abner Shimony, Volume One*, (Boston Studies in the Philosophy of Science 193), Boston: Kluwer Academic Publishers. (d) Bell, John S., 1964, "On the Einstein-Podolsky-Rosen Paradox", *Physics*, 1(3): 195–200. Reprinted in Bell 1987c: 14–21 and in Wheeler and Zurek 1983: 403–408. (e) Bohm, David and Basil J. Hiley, 1993, *The Undivided Universe: An Ontological Interpretation of Quantum Theory*, London: Routledge & Kegan Paul. (f) Aspect, Alan. (2015, December 16). Viewpoint: Closing the Door on Einstein and Bohr's Quantum Debate. *Physics* 8, 123.

To measure a closed quantum system we must also measure our experimental apparatus creating a larger system, but we can never look into this larger system undisturbed from the outside.

Heisenberg himself did not subscribe to Bohmian mechanics and instead, along with Niels Bohr, founded the Copenhagen interpretation of quantum mechanics. The purpose here is not to debate quantum theories. Our purpose is twofold. First, it will be argued that Boyd's use of Heisenberg's Uncertainty Principle is valid regardless of whether we subscribe to the Bohmian interpretation or the Copenhagen.[29] (Because both theories make the same probabilistic predictions, many physicists choose not to interpret quantum phenomena and instead prefer to "shut up and calculate," as Richard Feynman quipped.) Second, the discussion clarifies the self-referential role of the observer.

Heisenberg's Copenhagen interpretation suggests that quantum objects like electrons do not have perfectly precise positions and velocities. Here, an electron propagates as a wave—it is in many places at once, and in technical terms, it is in a "superposition" of all probable states—until we measure the electron's position which "collapses the wave-function" and reveals its particle-like properties.[30] In other words, when we zoom into a specific point to see whether the electron is there or not, we not only lose focus of the electron's wave-like properties, we cause the electron to (randomly) choose a particular state from the probabilities represented by the wave. The electron was spread out in a wave but upon measurement it takes on particle-like properties and answers questions of position. The Copenhagen interpretation finds that when an observer measures the system, quantum objects localize from a wave (with no precisely defined position) to a particle (with no precisely defined velocity). Both the Copenhagen and Bohmian views invoke the observer "intruding" upon complementary aspects of the observed, thereby affecting its future. Physicist Steven Weinberg commented that "As much as we would like to take a unified view of nature, we keep encountering a stubborn duality in the role of intelligent life in the universe, as both subject and student."[31]

Regardless of interpretation, uncertainty emerges and reduces any predictions one can make about future states into probabilities. The more precisely we want to measure a particle's position, the less we can know about its velocity. Without both values, we cannot precisely predict the particle's future which would otherwise be fully determined by Schrödinger's equation. One example where we only have statistical knowledge is radioactive decay. If we have 20 grams of a radioactive element and the half-life is one year, then we can predict that 10 grams will decay the first year and 5 grams will decay in the next. However, we cannot say whether an individual atom will decay or not; in the example it has a 50% likelihood in each year.[32] Indeed, the randomness is genuine for radioactive decay in the

---

[29] James Hasik attributed Boyd's views of Heisenberg's Uncertainty Principle to the Copenhagen interpretation and suggested problems, but as always, Boyd seems to have written generally enough that we cannot pin him to one interpretation. For example, Boyd cited *Thirty Years That Shook Physics* by George Gamow which introduced both the Copenhagen view and a view of De Broglie-Bohm mechanics from before John Bell. Frans Osinga also responded to Hasik on whether Boyd reasoned by analogy. See Hasik, James. (2012, May). Beyond Hagiography: Theoretical and Practical Problems in the Works and Legacy of John Boyd. 2.4. Retrieved from http://www.jameshasik.com/files/20120515_problems_of_boyd.pdf. See Osinga in Oslen J.A. (eds.) *Airpower Reborn: The Strategic Concepts of John Warden and John Boyd*. (2015). Naval Institute Press.

[30] Baggot, Jim. (2011). *The Quantum Story: A History in*. Oxford University Press.

[31] Weinberg, Steven. *Scientific American*. 271, no. 4, pp. 44.

[32] "A Million Random Digits with 100,000 Normal Deviates." Santa Monica, CA: RAND Corporation, 2001.

sense that information about it cannot be compressed.[33] Uncertainty implies randomness in physics, which is axiomatic in the Copenhagen view and a result of our ignorance in the Bohmian.[34] Still, a probabilistic (as opposed to deterministic) view of quantum objects has proved highly accurate and useful in technologies governed by Schrödinger's equation because it translates to predictions about ensemble behavior. This break between our uncertain descriptions at the individual level and our accurate pattern predictions at the ensemble level is significant because we large-scale objects are indeed ensembles of quantum objects and obey classical mechanics (where uncertainty is either minimal or averaged out). And it is precisely when we are not observing and intruding at the individual level that quantum objects act predictably at the statistical level. The act of observation brings uncertainty to our description at the individual level and prevents us from evaluating the match-up between our concept and reality. We cannot predict which atom will decay at a given rate, or which photon will reflect at a given angle, but we can predict what proportion of atoms will have decayed and what proportion of photons will have reflected. Boyd concludes his section on Heisenberg by remarking on the self-referentiality of the quantum situation:

> "When intrusion is total (that is, when the intended distinction between observer and observed essentially disappears), the uncertainty values indicate erratic behavior. When intrusion is low the uncertainty values do not hide or mask observed phenomena behavior, nor indicate significant erratic behavior. In other words, the uncertainty values not only represent the degree of intrusion by the observer upon the observed but also the degree of confusion and disorder perceived by that observer."

Entropy

Boyd related the confusion and disorder perceived by the observer when making point predictions to entropy and the Second Law of Thermodynamics. Entropy is a unique aspect of natural law because it points to an "arrow of time."[35] By contrast, all the dynamic physical descriptions are time-reversible in the quantum, classical, and relativistic theories (i.e., the equations work the same way forwards to the future as they do backwards into the past). With infinite precision, time-reversible theories leave no room for confusion. We can "see" all past and future states of the system. Entropy, on the other hand, is an irreversible process towards disorder. And unlike quantum mechanics or relativity, entropy corresponds with many of the experiences we have in our daily lives. For example, when we add milk to coffee it evenly mixes and never spontaneously separates. In this case and many others we can see an irreversible process where the past is fundamentally different than the future; a closed system goes from a well-ordered state to a disorganized, messy state. For another example, when we boil water, we are putting energy into the system and it gains potential for doing work (like cooking pasta or

---

[33] G.J. Chaitin. *Information, Randomness and Incompleteness, Papers on Algorithmic Information Theory*, World Scientific, Singapore, 1990. See also Calude, Cristian S., ed. (2007), *Randomness and Complexity. From Leibniz to Chaitin*, World Scientific.

[34] Calude, Cristian S. (2005). "Algorithmic Randomness, Quantum Physics, and Incompleteness." In: Margenstern M. (eds.) Machines, Computations, and Universality. MCU 2004. Lecture Notes in Computer Science, Vol. 3354. Springer, Berlin. Heidelberg. See also C. Calude and J. Michael. (2005). "Is Quantum Randomness Algorithmic Random? A Preliminary Attack." Dinneen Department of Computer Science University of Auckland, New Zealand. The authors argue that Heisenberg's uncertainty implies algorithmic randomness (as described by Gregory Chaitlin, i.e., it cannot be decided using a universal self-delimiting Turing machine whether an apparently random string of numbers is indeed random) which, in turn, implies incompleteness (in the Gödelian sense).

[35] Layzer, David. (1975, December). The Arrow of Time. *Scientific American*.

generating steam for turbines). The tumultuous boiling of the water may appear disorganized, but it is really generating complex order in the form of convection. If we leave the system alone, the water disperses heat into its environment, not the other way. We never expect water at room-temperature to spontaneously draw in heat from the air and start boiling. Closed systems evolve irreversibly from an ordered state towards disorder, from a state with capacity for doing work to one where work cannot be drawn out of it without putting energy in. More succinctly, differentials in heat have potential for doing work and as entropy spreads the heat that potential is lost. Closed systems always evolve towards higher entropy, like our disorienting attempt to more precisely match a consistent—but necessarily incomplete—concept descriptions with real-world observations.

Arthur S. Eddington remarked, "The law that entropy always increases holds, I think, the supreme position among the laws of Nature… If someone points out to you that your pet theory of the universe is in disagreement with… the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation."[36] The confusion and disorder caused by Heisenberg's Uncertainty Principle is indeed related to entropy in both the Bohmian and Copenhagen views.[37] Both our descriptions of quantum systems and entropy are necessarily statistical. A closed system of particles moving about has many more ways to find itself disordered than ordered. From this, we may expect our world to inevitably move towards more disorder and lifelessness, but that is not what we observe. Indeed, the Earth continuously draws in energy from the Sun which animates the weather and brings organic life to ecosystems. Boyd recognized that the only way to overcome entropy—to generate negative entropy—is to import order from another system that is larger and better organized. Boyd reasoned that "From this law it follows that entropy must increase in any closed system—or, for that matter, any system that cannot communicate in an ordered fashion with some other systems or environments external to itself." For Boyd's system of building concepts and matching them to reality, this means importing order from a stronger concept that can make sense of unexplained facts. We cannot work within one static objective view of reality. Boyd put it all together:

> "What an interesting result! According to Gödel we cannot—in general—determine the consistency, hence the character or nature, of an abstract system within itself. According to Heisenberg and the Second Law of Thermodynamics any attempt to do so in the real world will expose uncertainty and generate disorder. Taken together, these three notions support the idea that any inward-oriented and continued effort to improve the match-up of concept with observed reality will only increase the degree of mismatch."

Boyd viewed the increasing disorder within a closed system as a control mechanism that excites us into creatively building new and more powerful concepts. The cycle endlessly drives towards ever more complex concepts and actions. For Boyd, the human mind effectively combats an increase in

---

[36] Arthur. S. Eddington, *The Nature of the Physical World* (Chapter IV), Cambridge, UK: Cambridge University Press, 1929.

[37] The reconciliation is particularly salient for the Copenhagen because "every act of observation is by its very nature an irreversible process." Heisenberg, Werner. (1958). *Physics and Philosophy*. New York: Harper and Row. In the Bohmian view, entropy results from deterministic equations because of our ignorance rather than, as Ilya Prigogine argues, resonance that breaks time symmetry. The author is very much biased towards Prigogine's interpretation of physics, which builds on the Bohmian embrace realism and *nonlocality*, but removes ignorance as the cause of irreversibility.

entropy. Though Boyd stopped here in "Destruction and Creation," he continued to expand his ideas over the next two decades to better describe negative entropy systems. He later wrote how "Living systems are open systems; closed systems are non-living systems."[38]

As an open system, the human mind creates new concepts and negative entropy. Julian Simon later recognized how the human mind is the *ultimate resource*.[39] Yet the physical brain is itself a highly ordered system, the product of evolution. Concept building, as an output of the brain, must result from completely natural processes of negative entropy. The human mind is also part of the human body, which continually draws energy from the environment and disperses entropy back into the environment. Erwin Schrödinger found that all life feeds on negative entropy.[40] Living systems do not violate the Second Law of Thermodynamics because even though they generate pockets of negative entropy, they export more entropy into the environment consistent with an increase in the overall entropy. Negative entropy systems openly interact with their environment but remain self-bounded and self-perpetuating.[41] The process of generating complex order in the natural world was pioneered by Ilya Prigogine who developed a theory of dissipative structures.

Entropy has generally been associated with waste in an otherwise reversible process, such as friction causing a pendulum's swinging motion to stop. Prigogine, however, showed that open systems can generate negative entropy, and indeed self-organization, when two conditions prevail. First, the open system must be far from equilibrium. Energy or matter from the environment must flow through the system causing an excited and unstable state. Processes near equilibrium tend to be stable and revert to equilibrium. When more energy flows through the system, pushing it further away from thermodynamic equilibrium, the system tries to recover itself. It must dissipate entropy faster, creating new structures to reach an equilibrium. The system acts deterministically as it moves away from equilibrium until it reaches a critical point where the previous pattern becomes unstable. At the critical point, the system suddenly "chooses" among the stable patterns, giving the system new properties. The new stable state can again be treated deterministically until it moves even further away from equilibrium when it reaches another critical point, and so forth. These points of probabilistic choice between stable states are called bifurcations, which come with increasing rapidity as more energy or matter flow through the system. As it bifurcates down one path of another, the system takes on more active and complex properties.

The second condition for dissipative structures relates back to our ideas on self-reference, or internal circularity, which in this case keeps the dissipative system coherent (or correlated) as it moves increasingly far from equilibrium. The system requires inputs, an intermediate state, and a final set of outputs. Further, along the path to the final outputs the intermediate state must generate additional inputs as a by-product which feeds back into the system. In other words, we need self-amplifying feedback effects. In chemistry and physics these chain reactive processes are pervasive. For example,

---

[38] Boyd, John. (1987). "The Strategic Game of ? and ?"

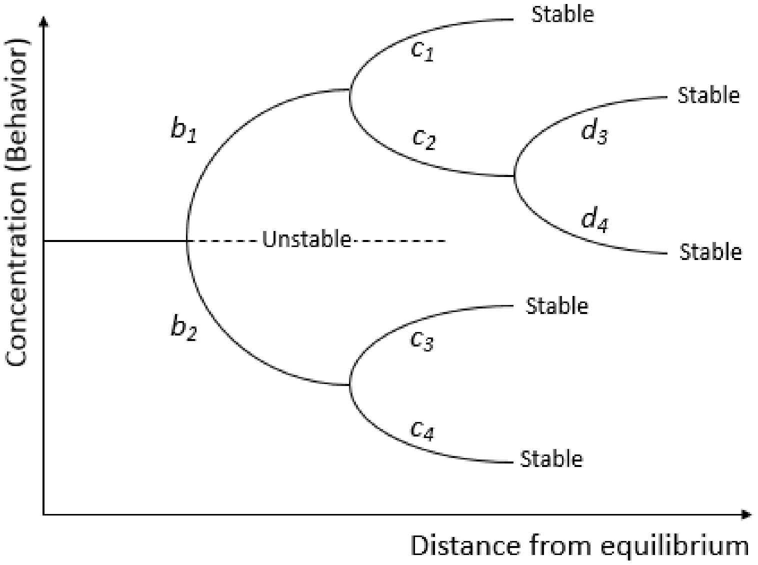
[39] Simon, Julian. (1983). *The Ultimate Resource*. Princeton, NJ: Princeton University Press.

[40] Schrodinger, Erwin. *What is Life?*

[41] Varela, Francisco J.; Maturana, Humberto R.; & Uribe, R. (1974). Autopoiesis: the organization of living systems, its characterization and a model. *Biosystems* Vol. 5, pp. 187–196

chemical reactions can produce heat which in turn increases the rate of reactions. Prigogine explained that we need "catalytic steps, such as the production of the intermediate compound Y from [input] compound X *together* with the production of X from Y. It is interesting to note that these conditions are satisfied in all living systems: Nucleotides code for proteins, which in turn code for nucleotides."[42] At far from equilibrium conditions, the feedback effects react to increasing instability that sparks bifurcation towards either new stable patterns or disintegration. Physical processes may then take on complex new behaviors and exhibit self-organization. As the environment becomes more complex, the system must have increasingly elaborate mechanisms to maintain itself. As Prigogine explained, "Bifurcations are the manifestation of an intrinsic differentiation between parts of the system itself and the system and its environment."[43]

To summarize, at far from equilibrium conditions, dissipative structures generate self-organization. We explained how the ordering process of negative entropy can occur when large amounts of energy or matter flow through a system. The system gains order at the expense of its environment, in which overall entropy increases. The dissipation itself does not explain self-organization in far from equilibrium conditions. After all, we have deterministic equations underlying dynamical systems, so irreversibility must come from somewhere. We then explained how circular feedback loops, where the system's inputs reference its own outputs, sparks sudden bifurcations which keep the system coherent and stable. These bifurcations bring about a place for irreversibility and the arrow of time. Feedback mechanisms lead to nonlinear effects, allowing systems to self-organize when far from equilibrium.



When nonlinear system moves far from equilibrium, resonances create bifurcations where new stable states of increasing complexity emerge. As distance from equilibrium increases, bifurcations become increasingly frequent.

Nonlinear Dynamics

We will dive just a bit deeper into the nature of bifurcations, and how they make nonlinear systems unpredictable, before reemerging to contrast the ideas of self-organization with the ideas of predictable control from the logical positivists. Linearity allows us to predict the long-term future and past because we do not encounter bifurcations where the system chooses among valid states. Linear systems and reversible processes tend to be idealizations, such as the frictionless pendulum. By contrast, nonlinear systems have several stable states. When a nonlinear system reaches a bifurcation point, we reach an irreversible process where the choice of state can only

---

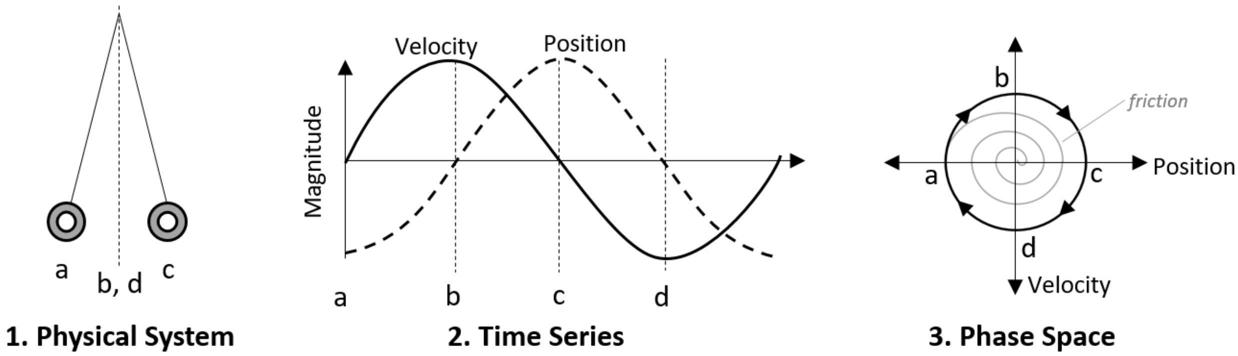
[42] Prigogoine, Ilya. (1996). *The End of Certainty.* New York, NY: The Free Press, pp. 66.
[43] Prigogoine, Ilya. (1996). *The End of Certainty.* New York, NY: The Free Press, pp. 69.

be described statistically. The real world regularly exhibits nonlinearity. One crucial aspect of nonlinear systems is that in most cases, they cannot be solved for (non-integrable). This was discovered by polymaths Jules-Henri Poincaré, who found that long term predictions cannot be made, even in fully deterministic systems, so long as nonlinearities prevail. Poincaré took Newton's famous equation for gravity and confirmed that indeed, the long-term future of a system of two bodies can be fully predicted with arbitrary accuracy using Newton's equations. However, when the influence of a third body is introduced, the system is no longer stable in most cases and long-term predictions cannot be made. The system becomes chaotic and many futures are possible. Similarly, you cannot unwind the system back into its history, several histories could have resulted in the present state of the system. It is illuminating to see how bifurcations, a term coined by Poincaré, are related to nonlinear systems.[44]

To first understand how Poincaré proved that the three-body problem was non-integrable, meaning there is no closed form solution from initial conditions, we must recognize that position and velocity are mathematically equivalent to frequency and time. That is, we can plot trajectories by wrapping frequencies around a coordinate system called phase space. For example, if we have a frictionless pendulum, we can plot the magnitude of the velocity as one frequency and position as another. When the pendulum swings through the center (from left to right) velocity hits a maximum and when its swing reaches its peak (maximal distance from the center), the velocity comes to zero. We can chart the possible states of the pendulum by plotting position on the x-axis and velocity on the y-axis of a coordinate system. Each moment in time will be represented by a point, and continuous time will be represented by a line. One full oscillation of the frequency corresponds to one rotation around the phase space graph of our example. In the case of the frictionless pendulum, we have a circle in phase space around a fixed point. And if we introduce friction, then the phase space portrait will cycle in toward a fixed point at the center because the pendulum loses velocity and the position stops at the center. In either case we have a fixed point—an "attractor"—in phase space that defines the natural state of the physical system.

---

[44] For more on the three-body problem, see Z.E. Musielak and B. Quarles. (2014). "The Three-Body Problem." *Reports on Progress in Physics*, Vol. 77, No. 6; and V. Szebehely (1990). "Chaos, Stability, and Predictability in Newtonian Dynamics." Published in Row, A.E. (eds.) *Predictability, Stability, and Chaos in N-Body Dynamical Systems*. New York, NY: Plenum Press, 1990.



(1) Each physical system, such as a pendulum, has a mode of motion. (2) The magnitude of the velocity and position can be plotted over time as separate frequencies. (3) For any given moment, the magnitude of the velocity and position can be plotted as coordinates in phase space. For the idealized pendulum, we have periodic motion around a fixed point, while the pendulum encountering friction spirals into a stationary state at the fixed point. This fixed point is called an attractor.

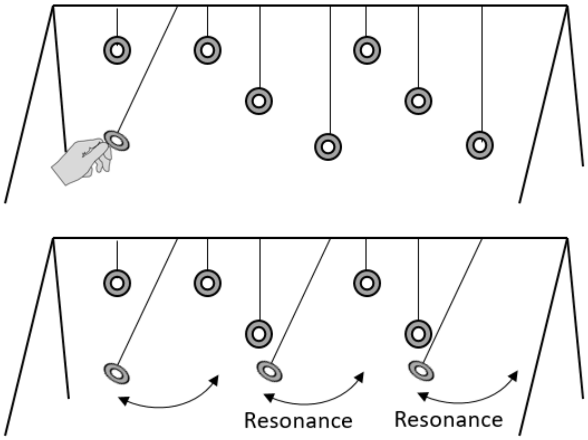
In the case of a pendulum, we are dealing with movement along a one-dimensional line with the need for only one position coordinate. To represent three-dimensional space, such as in the three-body problem of celestial orbits, we will need three position coordinates. Still, a three-dimensional body can be reduced to a single line in multi-dimensional phase space. We can then imagine plotting the evolution of three bodies, their position and velocity, on a single map of the combined system in phase space. Each of our three bodies is also affected by the others' gravity without physical interaction. This makes for a complicated set of 18 simultaneous equations. By calculating the net forces between the objects, we can again reduce the three-body system into a single point in phase space. Now, what makes the system non-integrable, and thus, unpredictable?

Remember, each mode of motion, such as a celestial orbit, corresponds to a frequency. Except for special cases, the frequencies that represent each body are not a sine wave of constant and periodic amplitude such as in the case of the idealized pendulum. Frequencies are affected by the changing influences of the other two bodies. The frequencies representing velocity and position morph as they move through phase space. Everything is still deterministic and time-reversible at this point. Over long enough timeframes, the nonlinear systems will generate a wide range of frequencies. Inevitably, two or more of the bodies will share the same frequency, corresponding to the same orbital period. At this point, we have resonance between frequencies. Resonance is the basic idea that when two objects share the same frequency, or share frequencies that are rational multiples of one another, they will drive each other to greater amplitudes. In our three-body problem, this manifests as large changes in position and velocity when orbits cross each other. Their gravitational forces lead to a pocket of phase space between them where force vectors are diverging everywhere (i.e., the bodies are repelling one another as though there were a source generating space between them). Once we encounter resonance, we get sudden qualitative changes in behavior leading to what Poincaré called bifurcations.[45] Closed form solutions require integrability for all system states. Yet resonance can lead to unbounded motion—infinities in phase space—making them non-integrable. We cannot predict

---

[45] Poincaré, Henri. (1885, September). "*L'Équilibre d'une masse fluide animée d'un mouvement de rotation*." *Acta Mathematica*, vol.7, pp. 259-380.

future or past states of systems when they encounter resonance. As time continues, one of our three erratic bodies will be ejected from the system due to resonance. We are left with a stable two-body system, and from this stable system we could not determine that its previous state had included three bodies. We arrive at an irreversible process. The resonant feature of dynamical systems is what makes them non-integrable and thus defines them as nonlinear. In short, resonance introduces outsized reactions and uncertainty to an otherwise deterministic system.



Suppose we hang weights at varying lengths from a common string. If we pull one of the weights and let it act as a pendulum, the other weights of the same length (corresponding to a particular frequency) will start swinging in sympathy. This is an illustration of resonance and nonlocality. Other weights whose string length is a rational multiple will also resonate, but not as strongly as the 1:1 case of same length. The system to the left has eight weights, and therefore has eight degrees of freedom, or eight directions of independent motion.

In special cases, long term solutions to the three-body problem can be found when we can linearize the problem, such as when the bodies are initially placed in a straight line or an equilateral triangle. But for most cases, linearization and inductive improvements (pioneered in the Kolmogorov-Arnold-Mosher Theorem) can only give us approximate solutions which become increasingly unsatisfactory as the forecast period progresses. More technically, we can take an average position and velocity of the bodies within phase space and use Fourier transformations to find the find the original frequencies of each body.[46] The process is what technicians use to filter out distinct sounds from an audio recording. The problem with linearization is that its approximations only hold under non-resonant conditions. When we initiate the system at time-zero, we can approximate what will happen after the first discrete moment, and feed that back into our linear approximation to get the second moment, and so forth. But when resonance is introduced, our linear calculations confront "dangerous" denominators that approaches zero, causing our answer to approach infinity.[47] At this point, approximations no longer hold and the system becomes unpredictable.[48] Individual trajectories, described by linear models, cannot be used because resonance brings nonlocal effects to the system.

---

[46] Wayne, Eugene C. (2008, January 22). "An Introduction to KAM Theory." http://math.bu.edu/people/cew/preprints/introkam.pdf. The Fourier transformation also gives us another view of the Bohmian interpretation of the Heisenberg Uncertainty Principle. See videos by 3Blue1Brown accessed on youtube.com, "But what is the Fourier Transform" and the follow-up video "The more general uncertainty principle, beyond quantum."

[47] Ilya Prigogine illuminates the dangerous denominators. Consider a system characterized by two frequencies. By definition, we have resonance whenever the sum $n_1\omega_1 + n_2\omega_2 = 0$, where n_1 and n_2 are nonvanishing integers and ω_1 and ω_2 are the frequencies. This means the ratio of the frequencies is a rational number, and in dynamic equations, we will have denominators with the term $(n_1\omega_1 + n_2\omega_2)$. Resonance causes the terms to diverge.

[48] The Lyapunov exponent determines when vectors exponentially diverge in area around our object, in this case due to resonance. Until they diverge, we can make reasonable predictions about the nonlinear system.

By now we have fully dismantled the reductionist view of logical positivism. Gödel proved that any logical system consistent with the real world is incomplete. There will be phenomena we cannot explain, or predict, using a unified system. When we try to ascertain whether any particular unpredictable fact is due to our ignorance of initial conditions rather than Gödel incompleteness, we run into Heisenberg's Uncertainty Principle. We are limited in the precision with which we can gather the positions and velocities necessary to make predictions at the individual level. We can only make statistical predictions. Quite separately from Heisenberg uncertainty, we have nonlinear systems in which we cannot make point predictions, *even when we know initial conditions with infinite precision.* Resonance leads to the non-integrability of most real-world systems because it "destroys" trajectory descriptions and by doing so, it introduces irreversible processes associated with entropy. When more energy or matter flows through a nonlinear system, moving it further away from equilibrium, resonances cause increasingly frequent bifurcations.

Bifurcations in nature can be interpreted as a manifestation of the system's effort to maintain itself, moving it towards new behaviors to export entropy at ever faster rates. From bifurcations we get turbulence, oscillating chemical reactions, and the seeds of life. Consider one hypothesis for the origins of life. Naturally occurring amino acids found on asteroids, when brought far from equilibrium in a low-angle impact with water, create the foundations for proteins.[49] For another example, neural frequencies (brainwaves) have natural resonances which tune to different stable states based on internal dynamics and the environment.[50] Perhaps more relevant to defense decision-making, turbulence is onset by resonance and bifurcations rather than a steady build-up of competing frequencies. Knowledge gained about turbulence is special, not universal. Information on the turbulence forming around the wing of a Boeing 707 has no relevance for an F-16 fighter.

If the three-body problem is worse than hard, then real systems on the order of 10^{23} particles are impossible. They require trial-and-error rather than prediction, regardless of computer power. Feynman wondered "Why should it take an infinite amount of logic to figure out what one tiny piece of space/time is going to do?"[51] Though the future cannot be fully predicted, we know that when matter is moved far from equilibrium it takes on new statistical properties and can move towards higher states of negative entropy. We may now speak of self-organizing behaviors that adapt to the environment.

Complex Adaptive Systems

It is important to note that unpredictable outcomes do not result from imperfect information or a lack of computing power. The very reason systems are unpredictable is also that which gives rise to complex order in the real world. Complexity arises in deterministic but nonlinear systems due to feedback loops, such as resonances between frequencies, in far from equilibrium conditions. Whether we are talking about atoms, photons, or the neurons in our brain, multiple independent components

---

[49] Steigerwald, Bill. (2012, March 9). "Meteorites Reveal Another Way to Make Life's Components." NASA's Goddard Space Flight Center. https://www.nasa.gov/topics/solarsystem/features/life-components.html.

[50] S. Atasoy, G. Deco, M. Kringelback & J. Pearson. (2017, September 1). "Harmonic brain modes: a unifying framework for linking space and time in brain dynamics."

[51] Gleick, James. (1987). *Chaos.* Auckland, NZ: Viking, pp. 121-156.

spontaneously correlate and work together in a coherent manner. Some complex systems that produce negative entropy, such as magnets and crystals, are self-organizing but not adaptive. Others, like our brain and Earth's ecosystem, produce a stationary state of on-going activity which is due to *persistent* interactions, feedback, and entropy dissipation.

While working on "Destruction and Creation" in 1976, John Boyd did not know how revolution in complexity would unfold. Ilya Prigogine's work on dissipative structures hadn't yet earned him a Nobel Prize. The Sante Fe Institute for complexity studies has not yet been founded. Despite evidence to the contrary, most scientists still viewed biological organization as the outcome of central direction rather than the unintended result of collective phenomena. Thinking had it that "founder" cells directed other cellular functions. Similarly, the queen ant was still thought to direct the many detailed activities of the ant colony. Across many disciplines, discoveries slowly chipped away at the core of the reductionist view represented by logical positivism. Systems cannot be understood through analysis. The future cannot be precisely predicted. Equilibrium conditions in linear systems are not of interest in the real world. John Boyd seemed to have grasped early on the importance of negative entropy systems, those that learn and adapt to a changing environment, and followed the scientific developments around the idea. These ideas coalesced around complex adaptive systems theory, which found forerunners in Norbert Wiener and Ludwig Bertalanffy. It has no relation whatsoever to the RAND method of systems analysis. Boyd continued to draw from a wide range of complexity studies until his death in 1996, as Frans Osinga thoroughly documented in *Science, Strategy, and War: The Strategic Theory of John Boyd*.

Though "complex adaptive systems" has become an umbrella term for a diverse range of research, we can briefly introduce it by contrasting it to so-called complicated systems. Complicated systems are generally man-made objects like tanks, airplanes, and satellites. They are made up of many parts, each of which may follow a complicated set of rules. If we study the characteristics of each part, we can fully describe the system. We can predict precisely how the complicated system will act under any circumstance. When we remove random parts from a complicated system its functioning abruptly stops. The lesson was hard learned after the failure of a single component caused the space shuttle Challenger to crash in 1986. While complicated systems are fragile and at best robust to shocks, they can still adapt. For example, the fly-by-wire system on the F-16 makes instantaneous adjustments to stabilize the aircraft. However, complicated systems are only adaptive to the extent that the range of possible environments and responses are enumerated. They cannot adapt to unforeseen circumstances. By contrast, complex adaptive systems are generally natural objects like the brain, ant colonies, and social networks. They are made up of many relatively simple parts working in parallel whose nonlinear interactions create novel system behaviors. Like the trajectories in our three-body problem, studying the individual parts cannot provide us an understanding of system behavior. When random parts are removed from complex systems they continue to function. Performance degrades slowly as more and more parts are removed because of redundant causal pathways. Complex systems can also adapt to new or unforeseen environments. They are not only robust to external shocks, such as in far from equilibrium conditions, but can benefit from those shocks. Internal feedback loops create novel behaviors required for complex systems to maintain themselves under unexpected conditions. Such environmental perturbations would quickly destroy a complicated system.

Matters of self-reference have been highlighted because of its asymmetric role in deductive and inductive systems. For deductive systems, self-referential feedback creates destructive limitations shown in the Russell paradox and Gödel incompleteness, elaborated on by Alan Turing with his halting problem for computing. Attempts to apply a consistent logical system to the real-world also encounter problems of precision. Initial conditions cannot be known with infinite precision due in one way or another to the self-referential presence of an observer. We need multiple frames of references simultaneously.[52] To avoid increasing disorientation, the observer must satisfy himself with statistical descriptions. For inductive systems, on the other hand, the "observer" is part of the system rather than distinct from the system. The observer cannot isolate the subject from their shared context to gain a controlled understanding of its function. All the system's parts act and react to one-another and the environment through resonant phenomena. Resonance is a form of feedback which creates nonlinearities and irreversible processes that can only be described statistically. As the system moves further from equilibrium, bifurcations are points where the system either evolves or dies. Feedback loops allow simple elements of matter to effectively coordinate by reacting to their neighbors rather than "waiting on orders from above."[53] Here, we see the creativity of bottom-up inductive processes.

Feedback loops play an important role in the self-organization of complex systems. Positive feedback, like resonance, propels the system forward. It is an essential attribute of Prigogine's concept for dissipative structures, providing context for bifurcations. Outputs are routed to inputs, creating an iterative and self-reinforcing process. Positive feedback leads to exponential growth, as seen in chain reactions, bacteria cultures, and embryos. Yet exponential growth is unsustainable in a finite and resource constrained environment. Systems with positive feedback also require a balancing force where variables move in opposite directions. Negative feedback loops provide such balance and slow the system down. Like a thermostat, it dampens positive feedback and other perturbations. Negative feedback pushes the system towards an equilibrium, towards an attractor in phase space. For most complex systems there are multiple equilibria which may be visited.

Feedback loops act along three general channels. First, as already discussed, outputs are routed to inputs in an iterative loop represented by exponential functions. Second, the macro-scale affects the micro-scale in a process called downward causation. Here, we recognize that systems are hierarchical. They have "integrative levels" such as a society being composed of people. In fact the word "society" has no meaning for a person in isolation. Integrative levels continue from people to organs, then tissues, cells, organelles, chemical compounds, atoms, and elementary particles. Reductionist accounts see only upward causality, that all system behaviors can be described by attributes of the most basic parts. A holistic view finds that higher levels of the system affect, and are affected by, lower level constituents. For example, societies constrain the actions and attitudes of

---

[52] The scientist requires experimentation on an isolated system, but such isolation is only a theoretical construct. In nonlinear systems, seemingly negligible interactions, such as the flap of a butterfly's wings or the existence of the observer, may have significant effects.

[53] Johnson, Steven. (2001). *Emergence*. New York, NY: Scribner, pp. 74.

the people as much as people contribute to societal behaviors. Similarly, the atomic structure downwardly affects the valence conditions of its electrons. A third channel of feedback is backward causation, where considerations about the future affect the present. Backward causation is most apparent in markets, where future expectations become embedded in today's price. The result may be the herd behavior of market bubbles and crashes, or the regulating behavior of arbitrage and entrepreneurship. Each of the three channels (iterative, downward causation, and backward causation) may propagate positive and negative feedback loops. The system effect is nonlinear in that outputs are not proportional to inputs.

Numerous feedback loops between decentralized parts help complex adaptive system build resilience to environmental perturbations. The importance of feedback loops was recognized by Norbert Wiener and W. Ross Ashby in the study of cybernetics. As Wiener's 1948 book explained, cybernetics is the science of control and communication in the animal and the machine.[54] Ashby argued that the internal regulation must have a requisite variety of mechanisms to deal with its environment characterized by continual flow and change.[55] As environmental challenges grow, the system needs to achieve a larger number of stable states to cope. Such variety requires a large the number of parts and numerous paths of communication. Donella Meadows explained how "Resilience arises from a rich structure of many feedback loops that can work in different ways to restore a system even after a large perturbation. A single balancing loop brings a system stock back to its desired state. Resilience is provided by several such loops, operating through different mechanisms, at different time scales, and with redundancy—one kicking in if another one fails."[56] Ludwig Bertalanffy described the idea of equifinality, how any outcome in an open system can have multiple nonlinear pathways of causality, creating resiliency.[57] By contrast, outcomes in closed systems have only one a single path of cause-and-effect. Naturally, with resilience comes inefficiencies associated with the maintenance of spare parts and idle feedback loops. "Redundancy *equals* insurance… The organism with the largest number of secondary uses is the one that will gain the most from environmental randomness and epistemic opacity!"[58]

Emergence

The scientific method relies on first isolating systems from their context and second decomposing them to an account of their parts. The problem with the first is that systems cannot be perfectly isolated. The issue becomes whether uncontrolled factors are negligible or significant. In nonlinear systems, even negligible causes can have an outsized impact, known as the "butterfly effect." We must also contend with the confounding role of the observer. Physicist David Bohm found that "The notion of a separate organism is clearly an abstraction, as is also its boundary. Underlying all this is

---

[54] Wiener, Norbert. (1948). *Cybernetics: Or Control and Communication in the Animal and the Machine.* Cambridge, MA: MIT Press.

[55] Ashby, W.R. (1962). "Principles of the Self-Organizing System". Published in H. Foerster and G.W. Zopf, Jr. (eds.), *Principles of Self-Organization*. US Office of Naval Research.

[56] Meadows, Donella. (2008). *Thinking In Systems*. Chelsea Green Publishing.

[57] Bertalanffy, Ludwig von. (1968), *General System Theory: Foundations, Development, Applications*. New York, NY: George Braziller.

[58] Taleb, Nassim. (2008). *The Black Swan*. Penguin. pp. 312-18.

unbroken wholeness even though our civilization has developed in such a way as to strongly emphasize the separation into parts."[59] The problem with the second, decomposition, is that systems are not made up of homogeneous and additive parts. Each integrative level of the system, from cosmology to humanity and down to the elementary particles, seems to require a different description. The reductionist believes that if the characteristics of the most elementary parts can be understood, then functions of the whole system can be predicted. Nonlinearities induced by feedback loops, however, destroys reductionism because it creates emergent properties at higher levels that cannot be predicted by lower level descriptions.

Emergence creates system behaviors which typically constrain the actions available to its parts. For example, convection is a phenomenon where liquid rotates in particular directions and cannot be explained from the interactions of an individual molecule. Similarly, consumers and suppliers in a marketplace make their own decisions about what to buy and sell, contributing to an emergent constellation of prices, which in turn conditions the choices of the market participants. James Gleick summarized the idea nicely. "Nonlinearity means that the act of playing the game has a way of changing the rules."[60] Downward causation contributes to strong emergence where accounts of the system cannot be reduced to the parts.

Emergent properties at higher levels are not formed outside the context of lower level interactions. The brain, for example, emerged from the interactions of lower level parts and co-evolved with the environment. Complex systems evolve from simple systems, creating a hierarchy of stable intermediate states. Hierarchies are pervasive in systems design because they provide resilience through multi-causal feedback loops and reduce the amount of information that any part of the system needs to process.[61] Each level in the hierarchy has qualitative differences, and yet they are integrated as a whole through a common process of formation. The hierarchy of levels share a self-affinity to one another. When we look at the tree's branch it shares many features as the whole, and when we look more closely at the twig it looks much like the branch, and so forth. In mathematics, self-affinity of structures at every scale is called fractal geometry.

One of the most surprising aspects of complex systems is that they are fractal in nature, meaning that they contain *infinite variety in finite form*. How can infinite variation reside within a finite physical space? Simple, through iterative processes just like feedback loops occurring over long periods of time. One famous example is the Koch snowflake. Imagine taking a triangle and adding triangles of one-third the length on each of its three sides. You now have a star with 12 sides. Then add triangles again reduced by a third to each side of the 12-sided star. You have 48 sides that is starting to look like a snowflake. Then continue adding smaller and smaller triangles on every side that is created from the last iteration. As you zoom further and further in, you see more and more detail. If you tried to

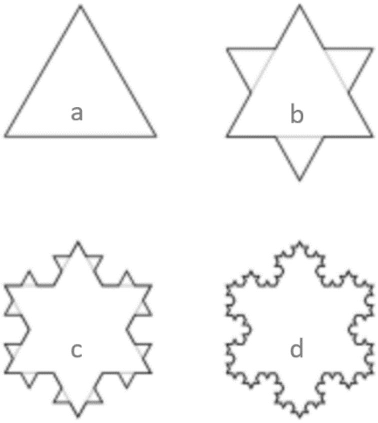
---

[59] D. Bohm & B.J. Hiley. (1995). *The Undivided Universe: An Ontological Interpretation of Quantum Theory*. Routledge. Also refer to Bohm's 1980 work, *Wholeness and the Implicate Order*. "Science itself is demanding a new, non-fragmentary world view… The notion that all these fragments are separately existent is evidently an illusion, and this illusion cannot do other than lead to endless conflict and confusion."

[60] Gleick, James. (1987). *Chaos*. Auckland, NZ: Viking.

[61] Meadows, Donella. (2008). *Thinking In Systems*. Chelsea Green Publishing, pp. 83-84.

measure the length of the Koch snowflake, you would find it infinitely long because every time you think you have a straight line to measure, it gets intersected and becomes longer. Yet the Koch snowflake takes up only a finite area in two-dimensions. The object is more than a one-dimensional line, but it is less than a two-dimensional plane. Its fractal dimension is about 1.26. The concept of fractals resulting from simple iterative processes is not just a mathematical curiosity, it is a pervasive feature of natural systems. Benoit Mandelbrot explained:



Koch snowflake, the first four iterations.

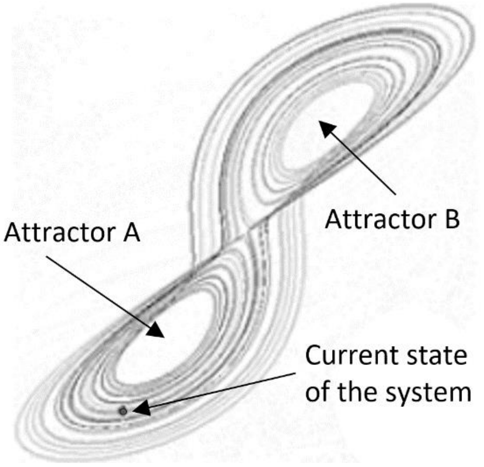
> "Fractal geometry has come to be viewed as 'natural.' It is used today for an improbably diverse set of tasks: compressing digital images over the Internet, measuring metal fractures, analyzing brain waves in an EEG machine, designing ultra-small radio antennae, making better optical cables, and studying the anatomy of lung bronchia.
>
> "The methods of fractal geometry have become part of the toolkit of fluid dynamics, hydrology, and meteorology. Its power comes from its unique ability to express a great deal of complicated, irregular data in a few simple formulae. This power is especially clear in the case of multifractality, which is fundamental in the study of turbulence and is also handy in financial markets."[62]

Fractals are driven by iterative recursion and are highly sensitive to initial conditions. Like nonlinear systems, nearly indistinguishable starting points lead to widely diverging outcomes. We therefore cannot predict the precise state a fractal structure. Yet through simulation they create stable patterns which never repeat themselves. If they did repeat, then they would settle into a predictable loop and information about it can be compressed. It is this idea of infinite variety in a finite space that allows a fractal system to generate the same general pattern without ever tracing over itself. The fractal shape has an infinitely complex boundary that defines it from its environment. Nevertheless, we can predict the rough shape of emerging patterns over time. Fractals have inherent order from iterating the same procedures, creating an *attractor* that defines the system's basic pattern of movement. Attractors with fractal geometry correspond to orderly patterns at all scales. The concept of an attractor provides insights into how some dissipative systems maintain coherence through bifurcation.

Consider our phase space graph of the idealized pendulum. Its velocity and position coordinates move in a repeating circle around a fixed point. The pendulum encountering friction spirals into that fixed point. The first example shows a periodic orbit around a fixed point, like two bodies orbiting around their center of gravity in a predictable system. The second example shows transient movement toward a fixed point, like a ball rolling to the bottom of a hill and stopping. In both cases, the fixed point attracts trajectory movements, such as a gravitational force, and defines its range of behaviors. In the examples, the attractor is the fixed point, but attractors don't need to be fixed. It can move continuously or non-continuously in simple or complex patterns. Consider the three-body system. Each body is attracted to two other bodies, causing non-periodic orbits around two erratic attractors until the system bifurcates due to resonance, ejecting one of the bodies. A single new attractor

---

[62] Mandelbrot, Benoit. (2006). *The Misbehavior of Markets: A Fractal View of Financial Turbulence*. Basic Books, pp. 117.



The three-dimensional Lorenz system shown above has two attractors. The orbit of the system state around the attractors never overlaps itself. Bifurcations occur when the system reaches a boundary condition and "chooses" an orbit around one attractor or another. The Lorenz system of equations arises in simple models of the atmosphere, lasers, dynamos, electric circuits, chemical reactions, and forward osmosis.

emerges. Now consider a complex system of persistent interactions among many bodies. Nonlinear systems generally produce multiple attractors that move around in phase space with infinite variety but always according to a stable pattern. When attractors have a fractal shape, it is called a *strange attractor*. The term was coined by David Ruelle and Floris Takens when describing the attractor resulting from turbulence in fluids.[63] The attractor moves through a defined region of phase space, it is stable, but never returns to the same point in phase space. In fact, the bifurcations resulting from resonance between frequencies in turbulent flows is what produces the evolving state of its strange attractor.

As explained above, bifurcations are a critical point where the system chooses among valid states, among attractors. When attractors have a complex fractal geometry, they provide the structure for complex emergent order to evolve. Strange attractors produce logically deep objects over long periods of time, representing the infinite variety in finite form. Consider an analogy. The military requirement for air superiority used to orbit around the piston engine. The cycle of aircraft development never revisited the exact same designs, but they revolved around the propeller technologies to generate power. All airframe designs had to conform to the constraints of the propeller. We then reached a critical point where the rise of jet engines became viable. There could have been various stable engine designs, but generally one becomes dominate. The cycle of aircraft development now orbits a new attractor until the next influx of energy and creativity develops another technological attractor. With the emergence of jet engines, propeller technologies are no longer a stable attractor and the system bifurcates, selecting a new stable attractor state. "While the dynamics of a chaotic system appear to have no pattern whatsoever, in reality, they conform to a remarkable fractal pattern, a strange attractor, which confines the system to a limited slice of state space and ensures that no state will ever repeat."[64] In this way, chaotic systems may evolve from the bottom-up in a completely stable pattern that nevertheless exhibits infinite complexity. When a system reaches a bifurcation point with multiple stable attractors emerging, the state of the system determines whether it is attracted to one or another. The boundary between points that evolve toward one attractor or another is infinitely complex. For a system state anywhere near the boundary conditions, we cannot predict which attractor it will end up in, often determined by random noise in the environment. These boundaries between attractors, like the famous Mandelbrot set, also exhibit fractal geometry at all scales. Yet over the distant future, as more bifurcations occur, system trajectories that have diverged to separate attractors will eventually

---

[63] Ruelle, David; Takens, Floris. (1971). "On the nature of turbulence". *Communications in Mathematical Physics*. 20 (3): pp. 167–192.

[64] Boeing, G. (2016). "Visual Analysis of Nonlinear Dynamical Systems: Chaos, Fractals, Self-Similarity and the Limits of Prediction". *Systems*. Vol. 4, no.4: pp. 37

return to being close together. This represents the stable global patterns emerging from strange attractors, even if the particular state at any given time appears locally unstable and unpredictable.[65]

Nearly all complex organizations in the nature have foundations in relatively simple and decentralized elements, but due to their nonlinear interactions, create stable emergent patterns around attractors. They build up from the bottom, and only then will emergent behaviors constrain the lower levels. The fact is equally true for economic marketplaces, which result from human action but *not from human design*. For the system to generate complexity, the parts must coordinate in a way that is beyond the information available to any individual part. Biological cells specialize based on what their neighbors are doing but end up with a functioning organism. A single ant could never assess the global situation of its colony, but by following the pheromones of its neighbors the colony thrives in a coordinated way. No model can direct a nation's resources to their most highly valued use, economic progress results from many individuals making separate plans and coordinating after-the-fact using the price mechanism. The theme is that systems generate complexity when relatively simple parts coordinate using local information only. They do not have order imposed on them independent of emergent properties. Perhaps unintuitively, simple systems give rise to complex behavior whereas complex systems give rise to simple behavior.[66] This is because nonlinearities create emergent properties that cannot be predicted. Complexity evolves from the bottom-up. On the other hand, predictability of response is often desirable for complicated systems like tanks, airplanes, and satellites. That being said, information on *future* technologies and environments cannot be held in one place, it is dispersed across all the people and institutions that engage in the larger system. We should almost certainly want our larger system of technology development, production, sustainment, and disposal to exhibit complex adaptive behaviors associated with bottom-up processes. The defense acquisition system is a weapon, but not like those tanks, airplanes, and satellites that emerge from its functioning. Researchers C.K. Biebracher, G. Nicolis, and P. Schuster summarized the viewpoint:

> "The maintenance of organization in nature is not—and cannot be—achieved by central management; order can only be maintained by self-organization. Self-organizing systems allow adaptation to the prevailing environment… We want to point out the superiority of self-organizing systems over conventional human technology which carefully avoids complexity and hierarchically manages nearly all technical processes."[67]

The purpose of this chapter has been to explain how complex order in the real world emerges from simple, iterative, systems of nonlinear interactions. The umbrella term of complex adaptive systems is used to describe self-organizing systems of emergent order that adapt to an uncertain environment. While these properties are not in general desirable for weapon systems that humans use in the field such as tanks, airplanes, and satellites, they are certainly desirable properties for the defense acquisition system as much as they are for market economies. Indeed, sustained technological progress cannot occur outside of a complex adaptive system. An analysis of quantitative natural laws cannot provide

---

[65] C. Grebogi, E. Ott, and J.A. Yorke. (1987). "Chaos, Strange Attractors, and Fractal Basin Boundaries in Nonlinear Dynamics." Science 238, no. 4827: pp. 632–638.

[66] Gleick, James. (1987). *Chaos*. Auckland, NZ: Viking.

[67] C.K. Biebracher, G. Nicolis, and P. Schuster. (1995). *Self-Organization in the Physico-Chemical and Life Sciences*. European Commission Report EUR 16456.

perfect foresight as to proper technological arrangements. When predictions fail, there are unforeseen events and no definite procedure can adapt to unforeseen events. It requires a different process of creativity and surprise resulting in new information. The human mind is one natural system that, unlike machines, can generate novel patterns for understanding higher levels of complexity.

Attributes of Complex Adaptive Systems

- *Self-organization*—Process where many local interactions create order without direction from above.
- *Feedback loop*—A circular process in which the system's output is returned or "fed back" to the system as input.
- *Nonlinearity*—Many possible responses are possible to a stimulus, and the cause and effect relationship is not evident.
- *Chaotic behavior*—Small changes in initial conditions can generate large changes in the outcome.
- *Stochastic*—Governed by chance. The behavior of a complex adaptive system can be inherently stochastic as elements of the system, the agents, can have randomness in their movement, and thus, in their interactions.
- *Attractors*—Catalysts that allow new behaviors to occur.
- *Inherent order*—Broad and complex outcomes resulting from local application of simple rules.
- *Emergent behavior*—New behavior represented by constant innovation and creativity
- *Context and embeddedness*—Systems reside within, and interacts with, other systems that influence it.
- *Porous boundaries*—The boundaries of the elements are blurry, allowing exchange and movement between them.
- *Co-evolution*—Progress occurs with constant tension and balance.

Adapted from Chaffee, Mary W and Margaret M McNeill. "A model of nursing as a complex adaptive system." Nursing Outlook *55 5 (2007): 232-241.*

Core concepts of complex adaptive systems were integrated into John Boyd's theories of human organization, leading him away from attrition warfare epitomized in World War I towards an idea of irregular maneuver warfare. Boyd found many predecessors of this form of thinking, from Sun-Tzu to Clausewitz to Liddell Hart. Strategic thinkers like Hans Delbruck and J.C. Wylie also investigated maneuver warfare. These philosophical trends toward thinking in terms of unpredictable, nonlinear systems coalesced into military doctrine in 1989 when Captain John Schmitt wrote the capstone doctrinal publication for the U.S. Marines titled *Warfighting*. For example, Schmitt wrote that

> "The very nature of war makes certainty impossible; all actions in war will be based on incomplete, inaccurate, or even contradictory information… While past battlefields could be described by linear formations and uninterrupted linear fronts, we cannot think of today's battlefield in linear terms… As a result, war is not governed by the actions or decisions of a single individual in any one place but emerges from the collective behavior of all the individual parts in the system interacting locally in response to local conditions and incomplete information. A military action is not the monolithic execution of a single decision by a single entity but necessarily involves near-countless independent but interrelated decisions and actions being taken simultaneously throughout the organization. Efforts to fully centralize military operations and to exert complete control by a single decisionmaker are inconsistent with the intrinsically complex and distributed nature of war."[68]

While complexity theories have penetrated the philosophy of military operations, attempts to translate the ideas into acquisition policy are few. Like combat, the development and deployment of technologies is an inherently uncertain and nonlinear process. Central direction by one or a small set of individual minds cannot generate the enormous complexity required for constant progress. The lesson was dramatically learned with the failure of socialist economies the world over. While apologists continue to dream of computing machines that will prevail over the seemingly chaotic and redundant coordination of the market economy, the impossibility of such a dream appears to be deeply built into

---

[68] (1997, June 20). "MCDP-1: Warfighting." U.S. Marine Corps.

the structure of our universe. The only realistic way to generate a system that exhibits complex behaviors beyond the foresight of any individual is to build from the bottom-up according to simple rules. Tacit coordination based on local conditions can then give rise to emergent order.